

Graduate Seminar on Algorithms and Optimization (S4C3) Discrepancy Theory

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Discrepancy Overview

- Given m sets $S_1, \dots, S_m \subseteq [n]$, color all elements of the ground set $[n]$ into red and blue, such that every set contains roughly an equal number of reds and blues.
- More specifically, we want $\min_i |\#\{\text{reds in } S_i\} - \#\{\text{blues in } S_i\}|$ to be as small as possible.
- Random coloring gives $O(\sqrt{n \log n})$.
- Spencer's result gives $6\sqrt{n}$, but non-constructive!

Discrepancy Overview

- General question: Random construction usually suffers from union bounds. How to avoid that and get better?
- Can we design polynomial time (deterministic) algorithm for them?
- How to apply them?
- Usual techniques: convex geometry + concentration

Classic Results

- 1 "Integer-making" theorem (Beck, Fiala, '81) and Six standard deviations suffice (Joel Spencer, '85) 1 student
- 2 Balancing vectors and Gaussian measures of n -dimensional convex bodies (Wojciech Banaszczyk, '98) 1 student
- 3 General questions: given matrix $A \in \mathbb{R}^{m \times n}$, find $\sigma \in \{-1, 1\}^n$ such that $\|A\sigma\|_\infty = \|\sum_i A_i \sigma_i\|_\infty$ is as small as possible.
- 4 Komlos Conjecture: If $\|A_i\|_2 \leq 1$ for any i , there exists $\sigma \in \{-1, 1\}^n$ such that $\|A\sigma\|_\infty = O(1)$.

Constructive Results

- 1 Constructive discrepancy minimization by walking on the edges (Lovett, Meka, '15) 1 student
- 2 Constructive discrepancy minimization for convex sets (Thomas Rothvoss, '17) 1 student
- 3 The Gram-Schmidt walk: a cure for the Banaszczyk blues (Bansal, Dadush, Garg, Lovett, '19) 1 student
- 4 A Unified Approach to Discrepancy Minimization (Bansal, Laddha, Vempala, '22) 1 student

Applications

- 1 Better bin packing approximations via discrepancy theory (Thomas Rothvoss, '16) 1 student
- 2 Flow Time Scheduling and Prefix Beck-Fiala (Bansal, Rohwedder, Svensson, '22) 1 student
- 3 Quasi-Monte Carlo Beyond Hardy-Krause (Bansal, Jiang, '25) 1 student

Other Settings

1 Matrix discrepancy

- Balancing matrices: Given (symmetric) matrices $A_1, \dots, A_n$, find $\sigma \in \{-1, 1\}^n$ such that $\|\sum_i A_i \sigma_i\|_{op}$ is as small as possible.
- Resolving Matrix Spencer Conjecture Up to Poly-logarithmic Rank (Bansal, Jiang, Meka, '23) 1 student
- Interlacing families II: mixed characteristic polynomials and the Kadison-Singer problem (Marcus, Spielman, Srivastava, '15) 1 student

2 Prefix Discrepancy, Smoothed Analysis, and Combinatorial Vector Balancing (Bansal, Jiang, Meka, Singla, Sinha, '22) 1 student

3 Optimal Online Discrepancy Minimization (Kulkarni, Reis, Rothvoss, '24) 1 student

Structure of seminars

Each seminar session is structured as follows:

① First part of the talk (10-20 minutes)

Introduce the topic of the talk.

Explain what the main goal or main result will be.

Give some motivation and provide some context — why is the result interesting/relevant?

② Questions

One or two (multiple-choice) questions from the speaker to the audience.

Take questions from the audience.

Structure of seminars

Each seminar session is structured as follows:

- ① First part of the talk (10-20 minutes)
- ② Questions
- ③ Second part of the talk (55-65 minutes) Present proofs, but focus on the main ideas rather than detailed calculations.
- ④ Discussion
Questions from the audience.

Parts 1 and 3 must not take more than 75 minutes in total.

Recall definitions and results from previous talks when you use them.

What we expect

- Prepare a talk on your assigned topic, including questions for the audience.
- Prepare a 1-2 pages summary containing the most important results and definitions.
- Give an approval talk approximately 2-3 weeks before your talk.
- Participate actively in the discussions during the seminar.
- In addition to reading the assigned chapter, it might be necessary to look into some other chapters or papers.

Topic assignment and registration

- Website with these slides, papers, and assignment will be available at <https://www.algopt.uni-bonn.de/teaching-pages/winter-term-25-26/graduate-seminar-discrepancy>
- If you would like to participate, send an email to Wenzheng Li (wzli@uni-bonn.de) indicating your name and topic preferences, including at least 3 topics, by **Friday 31 July**.
- We will inform you by email about the assignment of topics.
- Each participant will also be assigned a supervisor, Matthias Kaul or Wenzheng Li, who can help with questions.