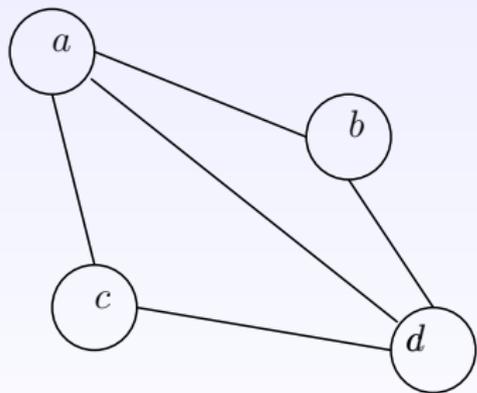# Hauptseminar Algorithmen und Optimierung (S2C2)
Spectral Graph Theory and Applications

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Spectral graph theory

Study the connection between properties of graphs and eigenvalues and eigenvectors of associated matrices.

$$M_G = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$L_G = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$$

What can the eigenvalues tell us about a graph?

- Connectivity: Second smallest eigenvalue of Laplacian > 0 if and only if G is connected.
- Conductance: edge density of cuts, related to the 2nd eigenvalue of normalized Laplacian.
- Colouring: Largest eigenvalue of adjacency matrix +1 is an upper bound on chromatic number.
- Random walks: mixing speed guided by 2nd eigenvalue of normalized adjacency matrix.

What can we do with the eigenvectors?

- Graph drawing
- Clustering
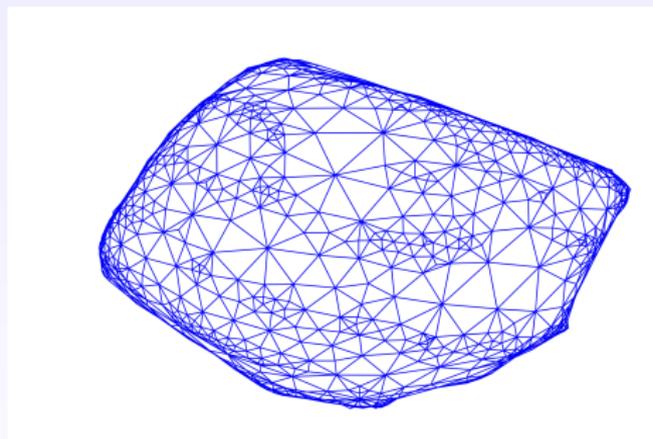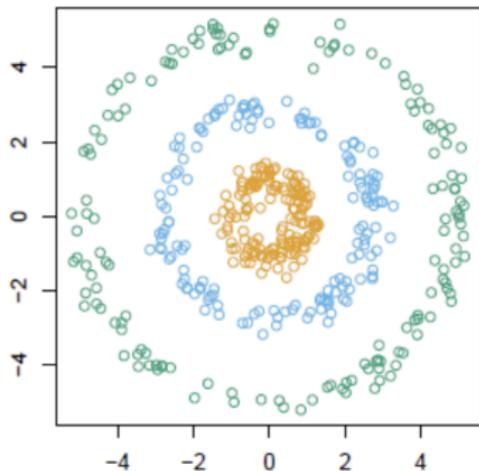
Laplacian systems and electrical flows

$$Lx = b$$

- Solutions to linear systems with Laplacian matrix correspond to electrical flows
- Linear systems can be solved in near linear time.
- Electrical flows can be used for graph sparsification
- Key ingredients for fast interior point methods for network flow problems.

Spielman: Spectral and Algebraic Graph Theory

- Draft manuscript with a gentle introduction, many examples, and broad coverage of topics
- We will cover only parts of the books
- 1-2 introductory lectures given by the course instructors

Structure of seminars

Each seminar session is structured as follows:

1 First part of the talk (10-20 minutes)
Introduce the topic of the talk.
Explain what the main goal or main result will be.
Give some motivation and provide some context — why is the result interesting/relevant?

2 Questions
One or two (multiple-choice) questions from the speaker to the audience.
Take questions from the audience.

Structure of seminars

Each seminar session is structured as follows:

1. First part of the talk (10-20 minutes)
2. Questions
3. Second part of the talk (55-65 minutes) Present proofs, but focus on the main ideas rather than detailed calculations.
4. Discussion
   Questions from the audience.

Parts 1 and 3 must not take more than 75 minutes in total.
Recall definitions and results from previous talks when you use them.

What we expect

- Prepare a talk on your assigned topic, including questions for the audience.
- Prepare a 1-2 pages summary containing the most important results and definitions.
- Give an approval talk approximately 2-3 weeks before your talk.
- Participate actively in the discussions during the seminar.
- In addition to reading the assigned chapter, it might be necessary to look into some other chapters or papers.

Proposed topic areas

1. Random walks, resistor networks, electrical flows
   Chapters 10–14, 3-4 students
2. Clustering and partitioning
   Chapters 20–23, 2-3 students
3. Laplacian solvers
   Chapters 32–38, 4-5 students
4. Planar graphs, drawing and colouring
   Chapters 15, 19, 24–25, 2-3 students

Topic assignment and registration

- Website with these slides, papers, and assignment will be available at `https://www.algopt.uni-bonn.de/teaching-pages/summer_term_26/copy_of_seminar_spectral_graph_theory`

- If you would like to participate, send an email to László Végh (lvegh@uni-bonn.de) indicating your name and topic preferences, by Wednesday 11 February noon.

- We will inform you by email about the assignment of topics.

- Each participant will also be assigned a supervisor, Wenzheng Li or Haoyuan Ma, who can help with questions.