Hauptseminar Algorithmen und Optimierung (S2C2) Convex Optimization

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Convex Optimization

- General optimization problems characterized by

$$\text{Local optima} = \text{Global optima}$$

- Rich theory on characterizing optimality and developing efficient algorithms
- Convergence bounds and polynomial-time algorithms can be developed for different classes of convex programs
- Generalizes linear programming
- Theory developed here can be used also in nonconvex settings.

Gradient descent

- Simple and natural iterative algorithm for $\min f(x)$

$$x^{(t+1)} = x^{(t)} - \alpha \nabla f(x^{(t)})$$

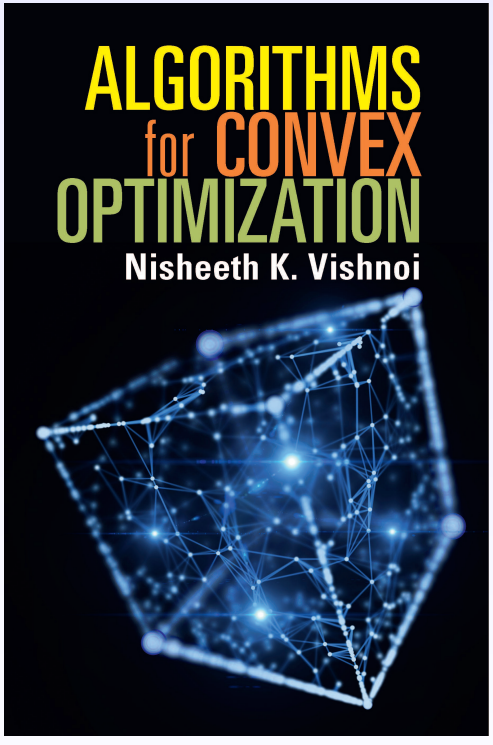
- Convergence speed depends on properties of the function.
- Types of convergence: get a ε-approximate solution in time proportional to $1/\varepsilon$, $1/\sqrt{\varepsilon}$, or $\log(1/\varepsilon)$ time.
- Can be extended to convex constrained setting $\min_{x \in K} f(x)$
- Needs limited access to the function
- Immense applications in machine learning

Newton's method

- Besides the gradient $f(x) \in \mathbb{R}^n$, can also access the Hessian $H(x) = [\partial_{ij} f(x)]_{i,j \in [n]}$
- Needs more computations than gradient descent but enables much stronger convergence bounds.
- Key to interior point methods that yield polynomial-time algorithms for linear and convex programs.

Vishnoi: Algorithms for Convex Optimization

- Gentle introduction to convex optimization
- Strong discrete optimization perspective: examples with flows and LP
- Main source for the seminar, but will have additional sources for certain topics



Structure of seminars

Each seminar session is structured as follows:

1. First part of the talk (10-20 minutes)
   Introduce the topic of the talk.
   Explain what the main goal or main result will be.
   Give some motivation and provide some context — why is the result interesting/relevant?
2. Questions
   One or two (multiple-choice) questions from the speaker to the audience.
   Take questions from the audience.

Structure of seminars

Each seminar session is structured as follows:

1. First part of the talk (10-20 minutes)
2. Questions
3. Second part of the talk (55-65 minutes) Present proofs, but focus on the main ideas rather than detailed calculations.
4. Discussion
   Questions from the audience.

Parts 1 and 3 must not take more than 75 minutes in total. Recall definitions and results from previous talks when you use them.

What we expect

- Prepare a talk on your assigned topic, including questions for the audience.
- Prepare a 1-2 pages summary containing the most important results and definitions.
- Give an approval talk approximately 2-3 weeks before your talk.
- Participate actively in the discussions during the seminar.
- In addition to reading the assigned chapter, it might be necessary to look into some other chapters or papers.

List of Chapters

1. Chapter 3: Convexity
2. Chapter 4: Convex Optimization and Efficiency
3. Chapter 5: Duality and Optimality
4. Chapter 6: Gradient Descent
5. (other): Gradient Methods for Constrained Optimization
   1–2 students

List of Chapters

6. Chapter 7/other: Mirror Descent and Multiplicative Weights Update
   1–2 students
7. (other): Online convex optimization
   2 students
8. Chapter 8: Accelerated Gradient Descent
9. Chapter 9: Newton's method
   2 students
10. Chapters 10-11: Interior Point Methods
    1-2 students

Topic assignment and registration

- Website with these slides, papers, and assignment will be available at https://www.algopt.uni-bonn.de/teaching-pages/winter-term-25-26
- If you would like to participate, send an email to Ulrich Brenner (brenner@or.uni-bonn.de) indicating your name and topic preferences, including at least 3 topics, by Monday 21 July 9am.
- We will inform you by email about the assignment of topics.
- Each participant will also be assigned a supervisor, Wenzheng Li or Haoyuan Ma, who can help with questions.